High Energy Physics

Lecture 9

In the previous lecture we have introduced the transition amplitude T_{fi}

$$T_{fi} = -i(2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) M$$

where M is the matrix element of the interaction potential:

$$-iM = \left[i\, j^{\mu}(a) \right] \left[-\frac{ig_{\mu\nu}}{q^2} \right] \left[i\, j^{\nu}(b) \right]$$

To get the transition probability we must take the mod-square of T and hence also the mod-square of M:

$$|M|^2 = \frac{1}{q^4} \left[j^{\mu *}(a)\, j^{\nu}(a) \right] \left[j^{*}_{\mu}(b)\, j_{\nu}(b) \right]$$

and we noticed that this could be written as the convolution of two tensors. After spin summation we obtained

$$|\overline{M}|^2 = \frac{1}{q^4} \overline{L}^{\mu\nu}_{a} \overline{L}^{b}_{\mu\nu}$$

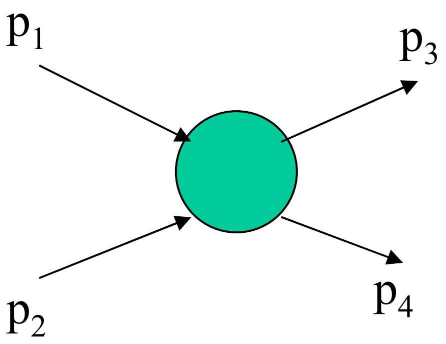
with

$$\boxed{\overline{L}^{\mu\nu} = 2e^2 \left[p_1^{\mu} p_3^{\nu} + p_1^{\nu} p_3^{\mu} + \frac{1}{2} q^2 g^{\mu\nu} \right]}$$

and hence

$$\left|\bar{M}\right|^2 = \frac{8e^4}{q^4}\left(p_1\cdot p_2 p_3\cdot p_4 + p_1\cdot p_4 p_3\cdot p_2 - m_a^2 p_2\cdot p_4 - m_b^2 p_1\cdot p_3 + m_a^2 m_b^2\right)$$

It is usual to define the following invariants (***Mandelstam variables***):



$$s = \left(p_1 + p_2\right)^2 = \left(p_3 + p_4\right)^2$$

$$t = \left(p_1 - p_3\right)^2 = \left(p_2 - p_4\right)^2$$

$$u = \left(p_1 - p_4\right)^2 = \left(p_2 - p_3\right)^2$$

By 4-momentum conservation we get

$$s + t + u = 2m_a^2 + 2m_b^2$$

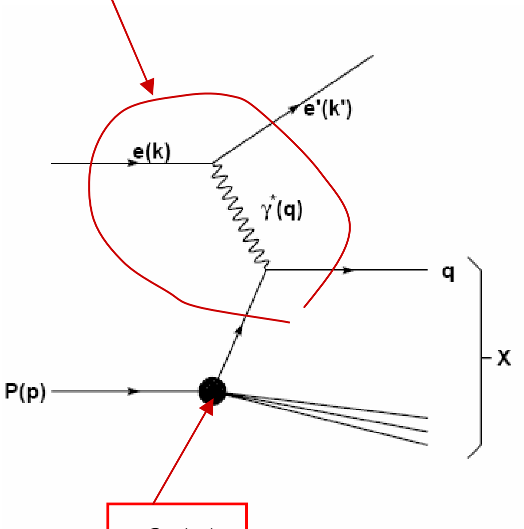
In terms of s, t, u and neglecting the masses (*ultra relativistic approximation*):

$$\left|\bar{M}\right|^2 = \frac{4e^4}{t^2}\left(s^2 + u^2\right)$$

and hence the differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{s}\frac{s^2 + u^2}{t^2}$$

hard sub process



$f_q(x)$

The differential cross section we have calculated is the diff cs of the ***hard sub process***.

To get the diff cs of the DIS process, this must be convoluted with the probability $f_q(x)$ to find a quark of flavour q that carries a fraction x of the proton momentum.

Thus we get the final formula

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2}{xyQ^2}\left\{\left(1 - y - \frac{x^2y^2M^2}{Q^2}\right)F_2(x) + xy^2F_1(x)\right\}$$

with $F_1(x) = \frac{1}{2}\sum_q e_q^2 f_q(x)$ and $F_2(x) = \sum_q e_q^2 x f_q(x)$

and we note that

$$F_2(x) = 2xF_1(x)$$

(***Callan-Gross*** relation)

The sums over q extend over all quark and antiquark flavours. Therefore …

… we can write in detail:

$$F_2(x)=\frac{4}{9}\left(xu(x)+x\overline{u}(x)\right)+\frac{1}{9}\left(xd(x)+x\overline{d}(x)\right)+\frac{1}{9}\left(xs(x)+x\overline{s}(x)\right)+\ldots$$

$$u=u_v+u_s;\quad d=d_v+d_s;$$

$$\overline{u}=u_{s;}\quad \overline{d}=d_s;\quad \overline{s}=s$$

$$F_2(x)=\frac{4}{9}\left(xu_v(x)+2x\overline{u}(x)\right)+\frac{1}{9}\left(xd_v(x)+2x\overline{d}(x)\right)+\frac{2}{9}xs(x)+\ldots$$

The parton densities are defined as the probabilities to find a parton with fractional momentum ***x***. We normalise the quark densities such that the valence quark densities, integrated over ***x*** from 0 to 1, are equal to the number of valence quarks in the proton:

$$\int_0^1 u_v(x)\,dx=2;\quad \int_0^1 d(x)\,dx=1$$

The parton densities can be defined also for the neutron. The neutron has one valence ***u*** quark and 2 valence ***d*** quarks, hence

$$u_v^n(x) = d_v(x); \quad d_v^n(x) = u_v(x)$$

The quark sea is reasonably assumed to be equal to the quark sea in the proton.

Therefore we have the following neutron structure function:

$$F_2^{en}(x) = \frac{4}{9} x d_v(x) + \frac{1}{9} x u_v(x) + Sea$$

If for the moment we neglect the strange quark sea (and charmed sea etc.), then we have the following proton and neutron structure functions:

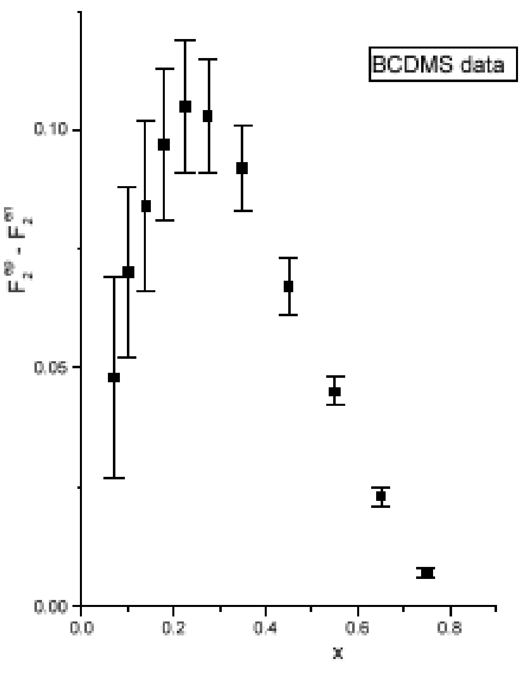
proton: $$F_2^{ep}(x) = \frac{4}{9} x u_v(x) + \frac{1}{9} x d_v(x) + \frac{10}{9} x S(x)$$

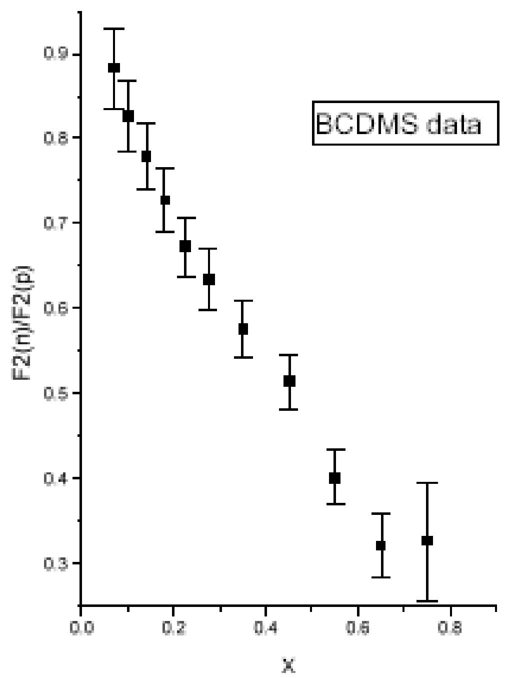
neutron: $$F_2^{en}(x) = \frac{4}{9} x d_v(x) + \frac{1}{9} x u_v(x) + \frac{10}{9} x S(x)$$

From these equations we can get the difference and the ratio of the proton and neutron structure functions:

$$F_2^{ep}(x) - F_2^{en}(x) = \frac{1}{3} x u_v(x) - \frac{1}{3} x d_v(x)$$

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{x u_v(x) + 4 x d_v(x) + 10 x S(x)}{4 x u_v(x) + x d_v(x) + 10 x S(x)}$$





Note the suppressed zero of the plot of the ratio F_2^{en}/F_2^{ep}.

We can see that for $\boldsymbol{x}\rightarrow 1$ the ratio tends to 0.25; this can be achieved most naturally if we assume that at $\boldsymbol{x}$ close to 1 the valence $\boldsymbol{u}$ quark density is dominant.

For $\boldsymbol{x}\rightarrow 0$ the ratio tends to 1: this is most easily explained by the dominance of the sea at small values of $\boldsymbol{x}$.

The shape of the difference indicates that the $\boldsymbol{u}$ quarks carry a greater fraction of the proton momentum than the $\boldsymbol{d}$ quarks at all values of $\boldsymbol{x}$, and especially near $\boldsymbol{x}$=0.25.

Next consider the following integral:

$$\varepsilon_q = \int_0^1 \left(x f_q(x) + x f_{\bar{q}}(x) \right) dx$$

it represents the fractional proton momentum carried by the quarks of flavour $\boldsymbol{q}$ and their antiquarks. Summed over all quark flavours we could expect this to …

… give the total (fractional!) momentum of the proton, *i.e.* we expect

$$\sum_q \varepsilon_q = \sum_q \int_0^1 \left(x f_q(x) + x f_{\bar{q}}(x) \right) dx = 1$$

If we understand the sum to run over all partons, those which are "seen" by the photon and those which are possibly "invisible", then this relation must be exact by definition; it is called the ***momentum sum rule***.

We can confront the sum rule with experiment.

To do this we assume that the proton and neutron consist only of u and d quarks and their antiquarks. Then we have

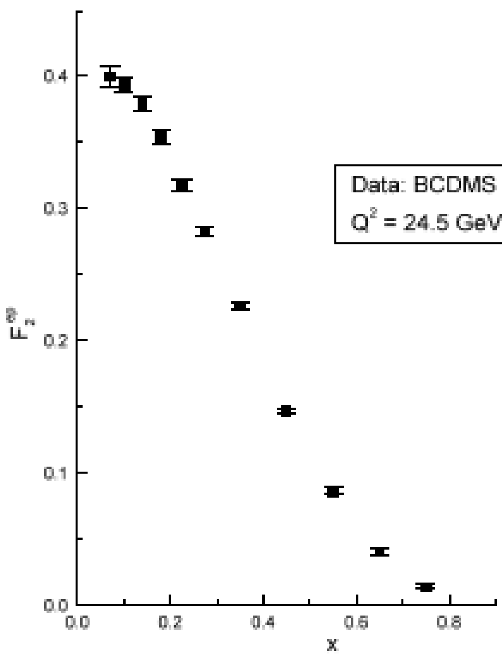
$$I_2^{ep} \equiv \int_0^1 F_2^{ep} dx = \frac{1}{9}\left(4\varepsilon_u + \varepsilon_d\right)$$

$$I_2^{en} \equiv \int_0^1 F_2^{en} dx = \frac{1}{9}\left(\varepsilon_u + 4\varepsilon_d\right)$$

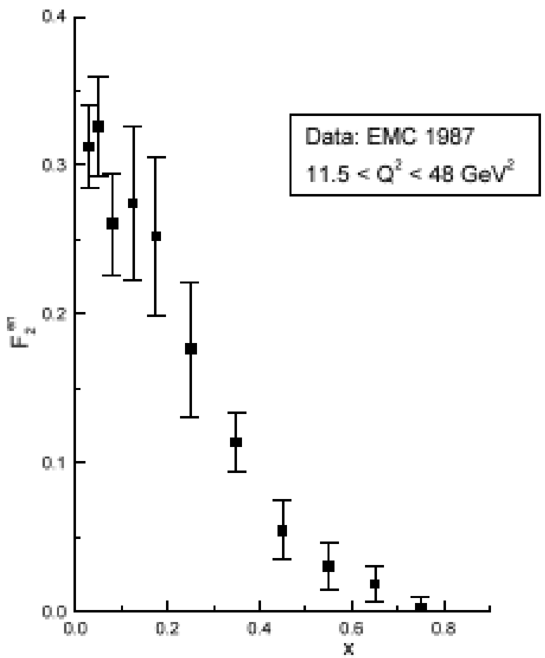
From experiment we have:

$$I_2^{ep} = 0.18 \quad \text{and} \quad I_2^{en} = 0.12$$

and hence $\varepsilon_u + \varepsilon_d = 0.54$



F_2^{ep} vs. x



F_2^{en} vs. x

In our calculation we have neglected the strange sea quarks. But we cannot accept the notion that the discrepancy between the momentum carried by the ***u*** and ***d*** quarks and the total momentum is the momentum of the strange sea since this would mean that the strange sea carries nearly half of the total momentum of the proton.

If we write the discrepancy in the form of

$$\varepsilon = 1 - \left(\varepsilon_u + \varepsilon_d \right)$$

then we get

$$\varepsilon = 0.46$$

Some of this may be carried by the strange sea, but most of it will be the contribution of partons ***invisible to the photon***. Such partons are identified with the ***gluons***, which are the ***gauge bosons of the strong interaction***.

The scaling violation which we have discussed in one of the previous lectures is the effect of the gluons.

We can learn more about the structure of the proton from neutrino DIS. So we turn now to the fascinating chapter of neutrino physics.